J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

FYJC FINAL EXAM - 01

SOLUTION SET

DURATION - 2 1/2 HR MARKS - 80

SECTION - I

Q1. Attempt ANY **SIX OF** THE FOLLOWING (12)

Q1

01. Using determinants show that the following set of points are collinear

A(3,1) ; B(4,2) ; C(5, 3)

SOLUTION

$$A = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 4 & 2 & 1 \\ 5 & 3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[3(2-3) - 1(4-5) + 1(12-10)\right]$$

$$= \frac{1}{2}\left[-3 + 1 + 2\right]$$

$$= 0$$

Hence the points are collinear

02. **Evaluate :** $\lim_{x\to 0} \frac{\log(1+6x)}{2x}$

SOLUTION

$$\lim_{x\to 0} \frac{\log(1+6x)}{2x}$$

$$= \lim_{x\to 0} \frac{6}{2}\,\frac{\log(1+6x)}{6x}$$

$$= 3(1) = 3$$

03. find the range of the given function : $f(x) = 9 - 2x^2$; $-5 \le x \le 3$

SOLUTION :

$$-5 \le x \le 3$$

$$0 \le x^2 \le 25$$

$$0 \le 2x^2 \le 50$$

$$0 \ge -2x^2 \ge -50$$

$$0 + 9 \ge 9 - 2x^2 \ge 9 - 50$$

$$9 \ge f(x) \ge -41$$

Range of f is $[-41 , 9]$

04. Find $\frac{dy}{dx}$ if $y = (x^2 + 4).(6x - 2)$

SOLUTION

$$y = (x^2 + 4).(6x - 2)$$

$$\frac{dy}{dx} = (x^2 + 4).\frac{d(6x - 2)}{dx} + (6x - 2)\frac{d(x^2 + 4)}{dx}$$

$$= (x^2 + 4).6 + (6x - 2)2x$$

$$= 6x^2 + 24 + 12x^2 - 4x$$

$$= 18x^2 - 4x + 24$$

05. Find centre and radius of the circle : $2x^2 + 2y^2 - 2x - 8y - 13 = 0$

SOLUTION

$$2x^2 + 2y^2 - 2x - 8y - 13 = 0$$

$$x^2 + y^2 - x - 4y - \frac{13}{2} = 0$$

On comparing with

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -1 \ ; \ 2f = -4 \ ; \ c = -13/2$$

$$g = \frac{-1}{2} \ ; \ f = -2 \ ; \ c = -13/2$$

$$C \equiv (-g , -f) \qquad R = \sqrt{g^2 + f^2 - c}$$

$$\equiv \left(\frac{1}{2}, 2\right) \qquad = \sqrt{\frac{1}{4} + 4 + \frac{13}{2}}$$

$$= \sqrt{\frac{1 + 16 + 26}{4}} = \frac{\sqrt{43}}{2}$$

06. Evaluate $\lim\limits_{x\to -2} \dfrac{x^7 + 128}{x^3 + 8}$

SOLUTION

$$= \lim_{x\to -2} \frac{x^7 - (-128)}{x^3 - (-8)}$$

$$= \lim_{x\to -2} \frac{x^7 - (-2)^7}{x^3 - (-2)^3}$$

Divide numerator and denominator by $x - (-2)$

$x \to -2$; $x \neq -2$ $\therefore$ $x - (-2) \neq 0$

$$= \lim_{x\to -2} \frac{\dfrac{x^7 - (-2)^7}{x - (-2)}}{\dfrac{x^3 - (-2)^3}{x - (-2)}}$$

$$= \frac{7}{3} \cdot \frac{(-2)^{7-1}}{(-2)^{3-1}}$$

$$= \frac{7}{3} \cdot \frac{(-2)^6}{(-2)^2}$$

$$= \frac{7}{3.} (-2)^4$$

$$= \frac{7}{3} (16)$$

$$= \frac{112}{3}$$

07. Find the length of latus rectum and equation of directrices of the ellipse

$$9x^2 + 8y^2 = 72$$

SOLUTION

$$9x^2 + 8y^2 = 72$$

$$\frac{9x^2}{72} + \frac{8y^2}{72} = 1$$

$$\frac{x^2}{8} + \frac{y^2}{9} = 1$$

$$a^2 = 8 \quad \therefore\ a = 2\sqrt{2}$$

$$b^2 = 9 \quad \therefore\ b = 3 \qquad b > a$$

Eccentricity

$$a^2 = b^2(1 - e^2)$$

$$8 = 9(1 - e^2)$$

$$\frac{8}{9} = 1 - e^2$$

$$e^2 = 1 - \frac{8}{9}$$

$$e^2 = \frac{1}{9}$$

✓ $e = {}^1/_3$

$$be = 3 \times {}^1/_3 = 1$$

$$\frac{b}{e} = \frac{3}{{}^1/_3} = 9$$

✓ eq. of directrices : $y = \pm\, {}^b/_e$

$$y = \pm 9$$

✓ length of latus rectum = $\dfrac{2a^2}{b} = \dfrac{16}{3}$

08. Prove : $\sin^2\left(\frac{\pi}{4} - x\right) + \sin^2\left(\frac{\pi}{4} + x\right) = 1$

SOLUTION

let

$\frac{\pi}{4} - x = \theta \quad \therefore\ x = \frac{\pi}{4} - \theta$

$\sin^2\left(\frac{\pi}{4} - x\right) + \sin^2\left(\frac{\pi}{4} + x\right)$

$= \sin^2\theta + \sin^2\left(\frac{\pi}{4} + \frac{\pi}{4} - \theta\right)$

$= \sin^2\theta + \sin^2\left(\frac{\pi}{2} - \theta\right)$

$= \sin^2\theta + \cos^2\theta$

$= 1$

Q2. (A) Attempt ANY **TWO OF** THE FOLLOWING (06)

Q2A

01.

Prove : $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}} = 2\cos\theta$

SOLUTION

LHS

$$= \sqrt{2+\sqrt{2+\sqrt{2(1+\cos 8\theta)}}}$$

$$= \sqrt{2+\sqrt{2+\sqrt{2\,.\,2\cos^2 4\theta}}}$$

$$= \sqrt{2+\sqrt{2+\sqrt{4\cos^2 4\theta}}}$$

$$= \sqrt{2+\sqrt{2+2\cos 4\theta}}$$

$$= \sqrt{2+\sqrt{2(1+\cos 4\theta)}}$$

$$= \sqrt{2+\sqrt{2\,.2\cos^2 2\theta}}$$

$$= \sqrt{2+\sqrt{4\cos^2 2\theta}}$$

$$= \sqrt{2+2\cos 2\theta}$$

$$= \sqrt{2(1+\cos 2\theta)}$$

$$= \sqrt{2\,.2\cos^2\theta}$$

$$= \sqrt{4\cos^2\theta}$$

$$= 2\cos\theta$$

02. Prove $$\frac{\sin A + \sin 5A + \sin 9A}{\cos A + \cos 5A + \cos 9A} = \tan 5A$$

SOLUTION

$$\frac{\sin 9A + \sin A + \sin 5A}{\cos 9A + \cos A + \cos 5A}$$

$$= \frac{2\sin\left(\frac{9A + A}{2}\right).\cos\left(\frac{9A - A}{2}\right) + \sin 5A}{2\cos\left(\frac{9A + A}{2}\right).\cos\left(\frac{9A - A}{2}\right) + \cos 5A}$$

$$= \frac{2\sin 5A \, . \cos 4A + \sin 5A}{2\cos 5A \, .\cos 4A + \cos 5A}$$

$$= \frac{\sin 5A\,\cancel{(2\cos 4A + 1)}}{\cos 5A\,\cancel{(2\cos 4A + 1)}}$$

$$= \tan 5A = \text{RHS}$$

03. Prove $$\cot^{-1}\left[8\right] + \cot^{-1}\left(\frac{9}{7}\right) = \frac{\pi}{4}$$

SOLUTION

$$\tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{7}{9}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{8} + \frac{7}{9}}{1 - \frac{1}{8} . \frac{7}{9}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{9 + 56}{72}}{\frac{72 - 7}{72}}\right)$$

$$= \tan^{-1}\left(\frac{65}{65}\right)$$

$$= \tan^{-1}(1)$$

$$= \pi/4$$

Q2. (B) Attempt ANY **TWO OF** THE FOLLOWING (08)

01. Find the coordinates of focus , equation of directrix , length of latus rectum and coordinates of ends of latus rectum of the parabola : $7x^2 + 16y = 0$

Q2B

SOLUTION :

Equation of parabola : $x^2 = -\frac{16}{7}y$

On comparing with : $x^2 = -4ay$, $4a = \frac{16}{7}$, $a = \frac{4}{7}$

Focus : $S \equiv (0,-a) \equiv (0,-4/7)$

Equation of directrix : $y - a = 0 \quad \therefore \; y - \frac{4}{7} = 0$

Length of latus recturm $= 4a = \frac{16}{7}$ units

Ends of latus rectum : $L \equiv (2a\ ,\ -a) \equiv (8/7,-4/7)$

$L' \equiv (-2a,\ -a) \equiv (-\ 8/7,-4/7)$

02. Find equation of circle having center at (1,4) and which cuts off a chord of length 6 units on the line $3x + 4y + 1 = 0$

SOLUTION

STEP 1 : $AP = PB = 3$ ($\perp$ from the centre bisects the chord)

STEP 2 :

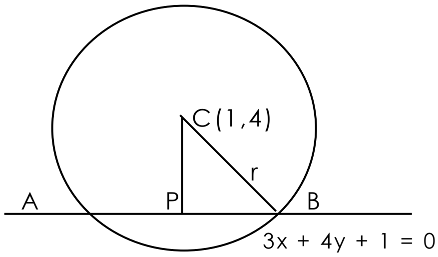
$$CP = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

$$= \left| \frac{3(1) + 4(4) + 1}{\sqrt{3^2 + 4^2}} \right|$$

$$= \left| \frac{3 + 16 + 1}{\sqrt{25}} \right|$$

$$= \left| \frac{20}{5} \right|$$

$$CP = 4$$



STEP 3 : In Δ CPB ; $CP^2 + PB^2 = r^2$

$$16 + 9 = r^2$$

$$r^2 = 25$$

$$r = 5$$

STEP 4 : $C(1, 4)$, $r = 5$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 1)^2 + (y - 4)^2 = 25$$

$$x^2 - 2x + 1 + y^2 - 8y + 16 = 25$$

$$x^2 + y^2 - 2x - 8y + 17 - 25 = 0$$

$$x^2 + y^2 - 2x - 8y - 8 = 0$$ ………. Equation of circle

03. Find equation of hyperbola passing through the point (−5,3) and having eccentricity √2

SOLUTION

Let the equation of the hyperbola be

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Since hyperbola passes through (−5,3) , it must satisfy the equation

$$\frac{25}{a^2} - \frac{9}{b^2} = 1 \quad \ldots\ldots \quad (1)$$

$$e = \sqrt{2} \quad \ldots\ldots \quad \text{given}$$

Now ; $b^2 = a^2(e^2 - 1)$

$b^2 = a^2(2 - 1)$

$b^2 = a^2$

subs in (1) $\frac{25}{a^2} - \frac{9}{a^2} = 1$

$a^2 = 16$

$\therefore\ b^2 = 16$

$\therefore$ the equation of the hyperbola is

$$\frac{x^2}{16} - \frac{y^2}{16} = 1$$

Q3. (A) Attempt ANY **TWO OF** THE FOLLOWING (06)

01. $2x - y + 3z = 9$, $x + y + z = 6$, $x - y + z = 2$ **Solve Using Cramer's Rule**

Q3A

SOLUTION

$$D = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 2 & -1 & 3 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} \end{array}$$

$= 2(1+1)+1(1-1)+3(-1-1)$
$= 2(2) + 1(0) + 3(-2)$
$= 4-6$
$= -2$

$$D_x = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 9 & -1 & 3 \\ 6 & 1 & 1 \\ 2 & -1 & 1 \end{vmatrix} \end{array}$$

$= 9(1+1)+1(6-2)+3(-6-2)$
$= 9(2) + 1(4) + 3(-8)$
$= 18 + 4 - 24$
$= 22 - 24$
$= -2$

$$D_y = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 2 & 9 & 3 \\ 1 & 6 & 1 \\ 1 & 2 & 1 \end{vmatrix} \end{array}$$

$= 2(6-2)-9(1-1)+3(2-6)$
$= 2(4) - 9(0) + 3(-4)$
$= 8-12$
$= -4$

$$D_z = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 2 & -1 & 9 \\ 1 & 1 & 6 \\ 1 & -1 & 2 \end{vmatrix} \end{array}$$

$= 2(2+6)+1(2-6)+9(-1-1)$
$= 2(8) + 1(-4) + 9(-2)$
$= 16 - 4 - 18$
$= -6$

$$x = \frac{D_x}{D} \; ; \quad y = \frac{D_y}{D} \; ; \quad z = \frac{D_z}{D}$$

$$= \frac{-2}{-2} \qquad = \frac{-4}{-2} \qquad = \frac{-6}{-2}$$

$$= 1 \qquad = 2 \qquad = 3$$

SS : {1,2,3}

02. $f(x) = 7x - 5$. find x satisfying $f(4x + 1) = f(3x - 2)$

SOLUTION

$f(4x + 1) = f(3x - 2)$

$7(4x + 1) - 5 = 7(3x - 2) - 5$

$7(4x + 1) = 7(3x - 2)$

$4x + 1 = 3x - 2$

$x = -3$

03. $y = \dfrac{x^2 + 3}{x\log x}$. Find $\dfrac{dy}{dx}$

SOLUTION

$$\frac{dy}{dx} = \frac{x.\log x \dfrac{d}{dx}(x^2 + 3) - (x^2 + 3) \dfrac{d}{dx}(x.\log x)}{(x.\log x)^2}$$

$$= \frac{x.\log x \,.\, 2x - (x^2 + 3).\left(x \dfrac{d}{dx}\log x + \log x \dfrac{d}{dx}x\right)}{(x.\log x)^2}$$

$$= \frac{2x^2.\log x - (x^2 + 3)\left(x \dfrac{1}{x} + \log x .1\right)}{(x.\log x)^2}$$

$$= \frac{2x^2.\log x - (x^2 + 3).(1 + \log x)}{(x.\log x)^2}$$

$$= \frac{2x^2.\log x - (x^2 + x^2.\log x + 3 + 3\log x)}{(x.\log x)^2}$$

$$= \frac{2x^2.\log x - x^2 - x^2.\log x - 3 - 3\log x}{(x.\log x)^2}$$

$$= \frac{(x^2 - 3).\log x - x^2 - 3}{(x.\log x)^2}$$

Q3. (B) Attempt ANY **TWO OF** THE FOLLOWING (08)

Q3B

01. Evaluate : $\lim_{x \to 0} \dfrac{(2^{\sin x} - 1)^3}{x.\tan x.\log(1+x)}$

SOLUTION

Dividing Numerator & Denominator by $\sin^3 x$, $x \to 0$, $x \neq 0$, $\sin x \neq 0$

$$= \lim_{x \to 0} \dfrac{\dfrac{(2^{\sin x} - 1)^3}{\sin^3 x} . \sin^3 x}{x . \tan x . \log(1 + x)}$$

Dividing Numerator & Denominator by x^3 , $x \to 0$, $x \neq 0$

$$= \lim_{x \to 0} \dfrac{\dfrac{(2^{\sin x} - 1)^3}{\sin^3 x} . \dfrac{\sin^3 x}{x^3}}{\dfrac{x . \tan x . \log(1 + x)}{x^3}}$$

$$= \lim_{x \to 0} \dfrac{\left(\dfrac{2^{\sin x} - 1}{\sin x}\right)^3 . \left(\dfrac{\sin x}{x}\right)^3}{\dfrac{\tan x}{x} . \dfrac{\log(1 + x)}{x}}$$

$$= \dfrac{(\log 2)^3 . (1)^3}{(1) . (1)}$$

$$= (\log 2)^3$$

02. the total cost of 't' toy cars is given by $C = 5(2^t) + 17$. Find the marginal cost and average cost at $t = 3$

SOLUTION :

Marginal Cost at t = 3

$= \dfrac{dC}{dt}$

$= 5(2^t.\log 2)$

Put $t = 3$

$= 5(2^3.\log 2)$

$= 5(8.\log 2)$

$= 40\log 2$

Average cost at t = 3

$= \dfrac{C}{t}$

$= \dfrac{5(2^t) + 17}{t}$

Put $t = 3$

$= \dfrac{5(2^3) + 17}{3}$

$= \dfrac{40 + 17}{3}$

$= \dfrac{57}{3}$

$= 19$

03. $y = \dfrac{\sec^3 x}{e^{4x}.(1+x)^5}$. Find dy/dx

SOLUTION :

STEP 1 :

$$\frac{d}{dx}\sec^3 x = 3\sec^2 x \,.\, \frac{d}{dx}\sec x$$

$$= 3\sec^2 x \,.\, \sec x \,.\, \tan x$$

$$= 3\sec^3 x.\tan x$$

STEP 2 :

$$\frac{d}{dx} e^{4x}.(1+x)^5$$

$$= e^{4x}.\frac{d}{dx}(1+x)^5 + (1+x)^5 \,.\, \frac{d}{dx} e^{4x}$$

$$= e^{4x}.\; 5(1+x)^4 \frac{d}{dx}(1+x) + (1+x)^5 \,.\, e^{4x} \frac{d}{dx} 4x$$

$$= e^{4x}.\; 5(1+x)^4 + (1+x)^5 \,.\, e^{4x} \,.\, 4$$

$$= 5.e^{4x}.\,(1+x)^4 + 4.e^{4x}.\,(1+x)^5$$

$$= e^{4x}.\,(1+x)^4 \left[5 + 4.(1+x)\right]$$

$$= e^{4x}.\,(1+x)^4 \;(9 + 4x)$$

STEP 3 :

$$\frac{dy}{dx} = \frac{e^{4x}.(1+x)^5 \frac{d}{dx}\sec^3 x - \sec^3 x \frac{d}{dx} e^{4x}.(1+x)^5}{\left[e^{4x}.(1+x)^5\right]^2}$$

$$= \frac{e^{4x}.(1+x)^5 \; 3\sec^3 x.\tan x - \sec^3 x \,.\, e^{4x}.\,(1+x)^4 \;(9+4x)}{\left[e^{4x}.(1+x)^5\right]^2}$$

$$= \frac{3e^{4x}.(1+x)^5 \sec^3 x \tan x - e^{4x}.(1+x)^4 (9+4x).\sec^3 x}{\left[e^{4x}\right]^2 \;(1+x)^{10}}$$

$$= \frac{e^{4x}.(1+x)^4 \sec^3 x \left[3(1+x)\tan x - (9+4x)\right]}{\left[e^{4x}\right]^2 \;(1+x)^{10}}$$

$$= \frac{\sec^3 x \left[3(1+x)\tan x - (9+4x)\right]}{e^{4x}.\;(1+x)^6}$$

SECTION - II

Q4. Attempt ANY **SIX OF** THE FOLLOWING (12)

01. 500 students appeared for an examination of whom 275 were boys . Out of 350 successful students , 150 were boys . Find number of unsuccessful girls

Q4

SOLUTION :

A ≡ student is a boy

B ≡ student successfully passed

	successful	B	β	TOTAL
Boy	A	$(AB) = 150$	$(A\beta) = 125$	$(A) = 275$
Girl	α	$(\alpha B) = 200$	$(\alpha\beta) = 25$	$(\alpha) = 225$
TOTAL		$(B) = 350$	$(\beta) = 150$	$N = 500$

unsuccessful girls $= (\alpha\beta) = 25$

02. The cost of living Index number for the years 1996 and 1999 are 140 and 200 respectively . A person earns Rs 11,200 per month in the year 1996 . What should be his earnings per month in the year 1999 , so as to maintain his former (i.e. of the year 1996) standard of living

SOLUTION :

Year	1996	1999
CLI	140	200
Income	□11200	?

1996

$$\text{Real income} = \frac{\text{Income}}{\text{CLI}} \times 100$$

$$= \frac{11200}{140} \times 100$$

$$= □\ 8000$$

1999

$$\text{Real Income} = \frac{\text{Income}}{\text{CLI}} \times 100$$

$$8000 = \frac{\text{Income}}{200} \times 100$$

$$\text{Income} = \frac{8000 \times 200}{100}$$

$$= □\ 16{,}000$$

03. Commodity

Commodity	P	Q	R	S	T
Base year price	12	28	x	26	24
Current year price	38	41	25	36	40

Find x if the Price Index Number by Simple Aggregate Method is 180

SOLUTION :

$$P_{01} = 180$$

$$\frac{\Sigma p_1}{\Sigma p_0} \times 100 = 180$$

$$\frac{180}{90 + x} \times 100 = 180$$

$$100 = 90 + x$$

$$x = 10$$

04. ${}^{n}P_3 : {}^{n-1}P_3 = 5 : 4$. Find n

SOLUTION :

$$\frac{{}^{n}P_3}{{}^{n-1}P_3} = \frac{5}{4}$$

$$\frac{\frac{(n)!}{(n-3)!}}{\frac{(n-1)!}{(n-1-3)!}} = \frac{5}{4}$$

$$\frac{\frac{n!}{(n-3)!}}{\frac{(n-1)!}{(n-4)!}} = \frac{5}{4}$$

$$\frac{n!}{(n-3)!} \times \frac{(n-4)!}{(n-1)!} = \frac{5}{4}$$

$$\frac{n!}{(n-3)!} \times \frac{(n-4)!}{(n-1)!} = \frac{5}{4}$$

$$\frac{n!}{(n-1)!} \times \frac{(n-4)!}{(n-3)!} = \frac{5}{4}$$

$$\frac{n(n-1)!}{(n-1)!} \times \frac{(n-4)!}{(n-3)(n-4)} = \frac{5}{4}$$

$$\frac{n}{n-3} = \frac{5}{4}$$

$$4n = 5n - 15$$

$$n = 15$$

05. A bag contains 10 white balls and 15 black balls . two balls are drawn in succession with replacement . What is the probability that first is white and second is black

SOLUTION :

exp : Two balls are drawn in succession without replacement

$E \equiv$ first is white AND second is black

$E \equiv A \cap B$

$P(E) = P(A \cap B)$

$P(E) = P(A) \times P(B|A)$

$= \frac{10}{25} \times \frac{15}{24}$

$= \frac{1}{4}$

06. Compute 5 yearly moving average values for the following data

Year	1997	1998	1999	2000	2001	2002	2003	2004
Sales(lacs)	2	4	6	8	13	12	14	11

SOLUTION :

Year	Sales	5 year moving Total (T)	5 year moving avg (T/5)
1997	2		
1998	4		
1999	6	33	6.6
2000	8	43	8.6
2001	13	53	10.6
2002	12	58	11.6
2003	14		
2004	11		

07. Find number of diagonals that can be formed in a 12 sided polygon

SOLUTION :

12 sided polygon

12 points

2 points define a line

$\therefore$ number of line that can be drawn

$= {}^{12}C_2 = 66$

But 12 are sides

$\therefore$ No . of diagonals $= 66 - 12$

$= 54$

08. mean of 10 items is 50 and standard deviation is 14 . Find sum of squares of all the items

SOLUTION :

$$\sigma^2 = \frac{\Sigma x^2}{n} - x^{2}$$

$$14^2 = \frac{\Sigma x^2}{10} - 50^{2}$$

$$196 = \frac{\Sigma x^2}{10} - 2500$$

$$\Sigma x^2 = 26960$$

Q5. (A) Attempt ANY **TWO OF** THE FOLLOWING (06)

01. a bag contains 3 red & 2 white balls . A second bag contains 2 red & 4 white balls . One ball is selected at random from the first bag and transferred to the second bag . Then a ball is drawn at random from the second bag . Find probability that it is red ball

SOLUTION :

exp : One ball is selected at random from the first bag and transferred to the second bag . Then a ball is drawn at random from the second bag

Q5A

$E \equiv$ ball drawn is RED

$E \equiv E_1 \cup E_2$

Where

$E_1 \equiv$ Red ball is transferred from Bag 1 to Bag 2 AND a Red ball is drawn from bag 2

$E_1 \equiv A \cap B$

$P(E_1) = P(A \cap B)$

$P(E_1) = P(A) \times P(B | A)$

$= \frac{3}{5} \times \frac{3}{7}$

NEW CONFIGURATION OF BAG 2

3 RED ; 4 WHITE

$= \frac{9}{35}$

$E_2 \equiv$ White ball is transferred from Bag 1 to Bag 2 AND a Red ball is drawn from bag 2

$E_2 \equiv A \cap B$

$P(E_2) = P(A \cap B)$

$P(E_2) = P(A) \times P(B | A)$

$= \frac{2}{5} \times \frac{2}{7}$

NEW CONFIGURATION OF BAG 2

2 RED ; 5 WHITE

$= \frac{4}{35}$

Now ;

$E \equiv E_1 \cup E_2$

$P(E) = P(E_1 \cup E_2)$

$P(E) = P(E_1) + P(E_2)$ …….. E_1 & E_2 are MUTUALLY EXHAUSTIVE

$= \frac{9}{35} + \frac{4}{35}$

$= 13/35$

02. Find the number of ways in which the letters of the word 'FATHER' can be arranged . How many of these arrangements

a) begin with A and end with R b) consonants occupy even places

SOLUTION :

letters of the word 'FATHER' can be arranged in 6P6 = 6! Ways

a) first place can be filled by letter A in 1 way .

Last place can be filled by letter R in 1 way

Having done that ,

Remaining 4 letters can be arranged into the remaining 4 places in ${}^4P_4$ = 4! ways

By Fundamental principle of Multiplication ,

Total arrangements = 4! = 24

b) the 3 even places can be filled by any 3 out of the 4 consonants (F,T,H,R) in ${}^4P_3$ ways . Having done that , remaining 3 places can be filled by the remaining 3 letters in ${}^3P_3$ = 3! Ways

By Fundamental principle of Multiplication ,

Total arrangements = ${}^4P_3$ 3! = 144

03. if $\Sigma p_0q_0 = 700$, $\Sigma p_0q_1 = 900$, $\Sigma p_1q_0 = 1070$ & P01(M-E) = 140 . Find P01(P)

SOLUTION :

$$P_{01}(ME) = \frac{\Sigma p_1q_0 + \Sigma p_1q_1}{\Sigma p_0q_0 + \Sigma p_0q_1} \times 100$$

$$140 = \frac{1070 + \Sigma p_1q_1}{700 + 900} \times 100$$

$$2240 = 1070 + \Sigma p_1q_1$$

$$\Sigma p_1q_1 = 1170$$

$$P_{01}(P) = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100$$

$$= \frac{1170}{900} \times 100$$

$$= 230$$

Q5. (B) Attempt ANY **TWO OF** THE FOLLOWING (08)

Q5B

01. Find cost of living index number

SOLUTION :

Group	p_0	p_1	w	$I = \frac{p_1}{P_0} \times 100$	Iw
A	200	320	20	$\frac{320}{200} \times 100 = 160$	3200
B	400	420	14	$\frac{420}{400} \times 100 = 105$	1470
C	100	120	15	$\frac{120}{100} \times 100 = 120$	1800
D	40	60	18	$\frac{60}{40} \times 100 = 150$	2700
E	20	28	10	$\frac{28}{20} \times 100 = 140$	1400
			$\Sigma w = 77$		$\Sigma Iw = 10570$

$$CLI = \frac{\Sigma Iw}{\Sigma w} = \frac{10570}{77} = 137.3$$

02. SOLUTION :

A ≡ inoculated

B ≡ not attacked by Hepatitis

	B	**β**	**TOTAL**
A	(AB) = 431	(Aβ) = 5	(A) = 436
α	(αB) = 291	(αβ) = 9	(α) = 300
TOTAL	(B) = 722	(β) = 014	N = 736

$$Q = \frac{(AB)(\alpha\beta) - (A\beta)(\alpha B)}{(AB)(\alpha\beta) + (A\beta)(\alpha B)}$$

$$= \frac{(431)(9) - (5)(291)}{(431)(9) + (5)(291)}$$

$$= \frac{3879 - 1455}{3879 + 1455}$$

$$= \frac{2424}{5334} = 0.4544$$

ROUGH WORK

LOG CALC.
3. 3845
– 3. 7270
AL $\bar{1}$. 6575
0. 4544

COMMENT : there is a significant degree of positive association between inoculation and not attacked by Hepatitis . Hence we can say inoculation was effective in controlling hepatitis

03. The probability that a person stopping at a petrol pump will ask for petrol is 0.80 , the probability that he will ask for water is 0.70 and the probability that he will ask for both is 0.65. Find the probability that he will ask for

a) only water b) neither petrol nor water

SOLUTION :

A : person will ask for 'PETROL' $P(A) = 0.80$

B : person will ask for 'WATER' $P(B) = 0.70$

$A \cap B$: person will ask for 'BOTH' $P(A \cap B) = 0.65$

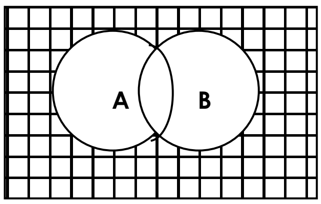
a) $E \equiv$ person will ask for either petrol or water

$E \equiv A \cup B$

$$P(E) = P(A \cup B)$$
$$= P(A) + P(B) - P(A \cap B)$$
$$= 0.80 + 0.70 - 0.65$$
$$= 0.85$$

b) $E \equiv$ person will ask for neither petrol nor water

$E \equiv A' \cap B'$



$$P(E) = P(A' \cap B')$$
$$= 1 - P(A \cup B)$$
$$= 1 - 0.85$$
$$= 0.15$$

Q6. (A) Attempt ANY **TWO OF** THE FOLLOWING (06)

01. The staff of a bank consists of Manager , the deputy manager and 10 other officers . A committee of 4 is to formed amongst them . Find the number of ways this can be done so as to include

a) I nclude the manager

Since manager is included , remaining 3 members have to be selected from the remaining 11 members (1 deputy manager + 10 officers) .

This can be done in $= {}^{11}C_3 = 165$ ways

Q6A

b) include the manger but not the deputy manager

since the manager is included but not the deputy manager , remaining 3 members have to be selected from the remaining 10 officers .

This can be done in $= {}^{10}C_3 = 120$ ways

c) neither the manager nor the deputy manager

since neither the manager nor the deputy manager is included , the 4 members have to be selected from the remaining 10 officers

This can be done in $= {}^{10}C_4 = 210$ ways

02. Find number of straight lines obtained by joining 10 points on a plane if 4 of which are collinear . Also find the number of triangles formed if 3 of them are collinear

SOLUTION :

10 points

2 points define a line

$\therefore$ number of line that can be drawn $= {}^{10}C_2 = 45$

But 4 points are collinear

Number of lines wrongly counted in these 4 collinear points $= {}^{4}C_2 = 6$ instead of 1

Hence

actual lines that can be drawn $= 45 - 6 + 1 = 40$

10 points

3 points define a triangle

$\therefore$ number of line that can be drawn $= {}^{10}C_3 = 120$

But 3 points are collinear

Number of triangles wrongly counted in these 3 collinear points

$= {}^{3}C_3 = 1$ instead of 0

Hence actual triangles that can be drawn $= 120 - 1 + 0 = 119$

03. Find the value of : $^{47}C_4 + \sum_{r=1}^{5} {}^{(52-r)}C_3$

SOLUTION :

$$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + \underline{{}^{47}C_3 + {}^{47}C_4}$$

$$= {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + \underline{{}^{48}C_3 + {}^{48}C_4}$$

$$= {}^{51}C_3 + {}^{50}C_3 + \underline{{}^{49}C_3 + {}^{49}C_4}$$

$$= {}^{51}C_3 + \underline{{}^{50}C_3 + {}^{50}C_4}$$

$$= \underline{{}^{51}C_3 + {}^{51}C_4}$$

$$= {}^{52}C_4$$

Q6. (B) Attempt ANY **TWO OF** THE FOLLOWING (08)

Q6B

01. Obtain trend line by method of least squares

Year	1977	1978	1979	1980	1981
No. of boxes (000)	20	19	21	24	25

SOLUTION :

t	y	u = t − 1979	u^2	yu
1977	20	−2	4	−40
1978	19	−1	1	−19
1979	21	0	0	0
1980	24	1	1	24
1981	25	2	4	50
	109	0	10	15
	Σy	Σu	Σu^2	Σyu

$$y = a + bu$$
$$\Sigma y = na + b\Sigma u$$
$$109 = 5a$$
$$a = 21.8$$

$$yu = au + bu^2$$
$$\Sigma yu = a\Sigma u + b\Sigma u^2$$
$$15 = b(10)$$
$$b = 1.5$$

Hence trend line , $y = 21.8 + 1.5u$ where $u = t - 1979$

02. Calculate Dorbish Bowley's Price Index number

SOLUTION :

p_0	q_0	p_1	q_1	p_1q_0	p_1q_1	p_0q_0	p_0q_1
8	20	11	5	220	55	160	40
7	10	12	10	120	120	70	70
3	30	5	20	150	100	90	60
2	50	4	15	200	60	100	30
				690	335	420	200
				Σp_1q_0	Σp_1q_1	Σp_0q_0	Σp_0q_1

$$P_{01}(L) = \frac{\Sigma p_1q_0}{\Sigma p_0q_0} \times 100$$

$$= \frac{690}{420} \times 100$$

$$= 164.3$$

$$P_{01}(P) = \frac{\Sigma p_1q_1}{\Sigma p_0q_1} \times 100$$

$$= \frac{335}{200} \times 100$$

$$= 167.5$$

$$P_{01}(D-B) = \frac{P_{01}(L) + P_{01}(P)}{2}$$

$$= \frac{164.3 + 167.5}{2}$$

$$= 165.9$$

03. 2 integers are selected at random from integers 1 to 11 . if the sum of the integers is even , find the probability that both the numbers are odd

SOLUTION :

exp : 2 integers are selected at random from integers 1 to 11 $\quad n(S) = {}^{11}C_2 = 55$

A : both the numbers are odd

$n(A) = {}^{6}C_2. = 15$; $\quad P(A) = 15/55$

B : sum of integers is even

In that case either both the integers need to be ODD or both need to be even

$n(B) = {}^{6}C_2 + {}^{5}C_2. = 15 + 10 = 25$; $\quad P(B) = 25/55$

$A \cap B$: sum of integers is even and both the numbers are ODD

$n(A \cap B) = {}^{6}C_2. = 15$; $\quad P(A \cap B) = 15/55$

$E \equiv$ both the numbers are odd if the sum of integers is even

$E \equiv A | B$

$P(E) = P(A | B)$

$= \dfrac{P(A \cap B)}{P(B)}$

$= {}^{15}/_{25}$

$= {}^{3}/_{5}$